Serie 04 - Solution

Preamble

PN-junction under bias

In the previous series, we derived the charge, electric field, and electric potential distributions in a PN-junction at thermal equilibrium. By applying a bias to this junction, these previously calculated distributions will change. Since the semiconductor is conductive outside the space charge region **(SCR)**, no electric field can exist in these regions. This implies that any potential applied to the PN-junction will directly affect the potential in the SCR. Essentially, we can rewrite the previous equations by replacing the built-in potential ϕ_b with the built-in potential ϕ_b minus the applied forward voltage V_F.

Given constants

$n_i(Si) = 1.5 \cdot 10^{10}\,[cm^{-3}] \quad @ \quad T = 300\,[K]$
$k = 8.62 \cdot 10^{-5}\,[eV/K]$
$q = 1.60 \cdot 10^{-19}\,[C]$
$\epsilon_0 = 8.85 \cdot 10^{-14}\,[F/cm]$
$\epsilon_{Si} = 11.7 \cdot \epsilon_0$

Exercise 01

We consider a PN-junction with known dopant concentrations of $N_A = 10^{16}\left[cm^{-3}\right]$ for the P-type region $N_D = 10^{15}\left[cm^{-3}\right]$ for the N-type region. At room temperature, with an applied forward bias voltage of $U = -5\,[V]$, find the depletion width and the maximum electric field.

Solution

The first step is to calculate the built-in potential:

$$\phi_b = \frac{kT}{q} \ln\left(\frac{N_a N_d}{n_i^2}\right) = 634\,[mV] \tag{1}$$

Next, we find the depletion width in the N-type region, X_n, by modifying the formula derived in **Exercice 02E** of **Serie 03**, following the adjustments described in the preamble:

$$X_n\left(V\right) = \sqrt{\frac{2\epsilon_{Si} N_a}{qN_d\left(N_a + N_d\right)}\left(\phi_b - V\right)} = 2.57\,[\mu m] \tag{2}$$

Similarly, we determine the depletion width in the P-type region, X_p. Alternatively, this length can also be derived using the space charge neutrality condition:

$$X_p\left(V\right) = \sqrt{\frac{2\epsilon_{Si}N_d}{qN_a\left(N_a+N_d\right)}\left(\phi_b - V\right)} = 257\left[nm\right] \tag{3}$$

The total depletion width, X_d, can be obtained either by summing the depletion widths of both regions or by directly using the formula for X_d, as demonstrated here:

$$X_d\left(V\right) = \sqrt{\frac{2\epsilon_{Si}}{q\left(N_a+N_d\right)}\left(\phi_b - V\right)} \cdot \left(\frac{\sqrt{N_a}}{\sqrt{N_d}} + \frac{\sqrt{N_d}}{\sqrt{N_a}}\right) \tag{4}$$

Which can be rewritten as:

$$X_d\left(V\right) = \sqrt{\frac{2\epsilon_{Si}\left(N_a+N_d\right)}{qN_aN_d}\left(\phi_b - V\right)} = 2.83\left[\mu m\right] \tag{5}$$

Finally, we calculate the maximum electric field E_0. There are two possible approaches:

Using Maxwell's equations and integrating the charge density over the depletion width of one of the two regions.

$$E_0 = \frac{qN_aX_p}{\epsilon_{Si}} = \frac{qN_dX_n}{\epsilon_{Si}} = 3.98\left[\frac{MV}{m}\right] \tag{6}$$

Or applying the modifications described in the preamble to the formula derived in the previous series:

$$E_0(V) = \sqrt{\frac{2qN_aN_d}{\epsilon_{Si}\left(N_a+N_d\right)}\left(\phi_b - V\right)} = 3.98\left[\frac{MV}{m}\right] \tag{7}$$

Note: Be very careful with units. The preferred unit in semiconductor devices is $[cm]$, not $[m]$.

Exercise 02

Find the concentrations of dopants N_a and N_d of a PN junction using the room-temperature Mott-Schottky plot given below. Assume that the N-type and P-type regions are doped only with their respective dopants. The slope of the graph is $\alpha = -1350\left[\left(\frac{\mu F}{cm^2}\right)^{-2} V^{-1}\right]$and the intersection with the horizontal axis is$V_0 = 855\,[mV]$.Your hypothesis is that the diode is asymmetrically doped $(N_a \gg N_d)$. Verify this hypothesis.

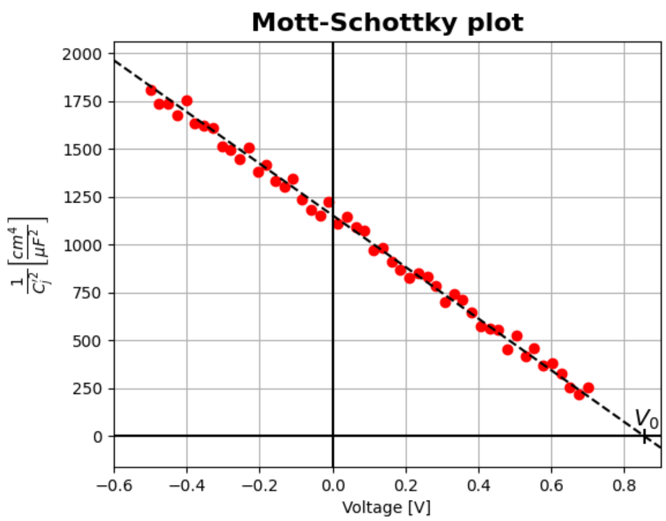

Figure 1: Mott-Schottky plot.

Solution

First, as developed during the course, we determine the depletion capacitance. As previously explained, the SCR does not contain any free charge carriers, making it behave as an insulator, while the N-type and P-type regions of the PN junction remain conductive. This allows us to model the structure as a planar capacitor, where the plate separation depends on the voltage, and the dielectric is silicon.

$$C = \frac{\epsilon}{d} \tag{8}$$

Thus, we use the formula for planar capacitance, substituting the depletion width equation:

$$C_j = \frac{\epsilon_{Si}}{\sqrt{\frac{2\epsilon_{Si}(N_a+N_d)}{qN_aN_d}\left(\phi_b - V\right)}} \tag{9}$$

The space-charge neutrality equation directly relates the depletion width to the dopant concentration:

$$N_d X_n(V) = N_a X_p(V) \quad \iff \quad \frac{X_n(V)}{X_p(V)} = \frac{N_a}{N_d} \tag{10}$$

In the case of high doping asymmetry, the total depletion width is primarily determined by the region with the lower dopant concentration. Therefore, we can express the following result:

$$X_d(V) \approx \begin{cases} X_n(V) & \text{if} \quad N_a \gg N_d \\ X_p(V) & \text{if} \quad N_d \gg N_a \end{cases} \tag{11}$$

Since in our case $N_a \gg N_d$, we can rewrite the junction capacitance as:

$$C_j \approx \frac{\epsilon_{Si}}{X_n(V)} = \frac{\epsilon_{Si}}{\sqrt{\frac{2\epsilon_{Si} N_a}{q N_d (N_a + N_d)} (\phi_b - V)}} \tag{12}$$

Rewriting the expression for the junction capacitance in the Mott-Schottky form:

$$\frac{1}{C_j^2} = (\phi_b - V) \frac{2}{q\epsilon_{Si}} \frac{N_a}{N_d (N_a + N_d)} \tag{13}$$

Since we are still considering an asymmetrically doped case, we apply the following approximation:

$$N_a \gg N_d \quad \Longrightarrow \quad N_a + N_d \approx N_a \tag{14}$$

Hence:

$$\frac{1}{C_j^2} \approx (\phi_b - V) \frac{2}{q\epsilon_{Si}} \frac{N_a}{N_d N_a} = (\phi_b - V) \frac{2}{q\epsilon_{Si} N_d} \tag{15}$$

From this formula, we clearly see that the curve follows a slope of:

$$\alpha = -\frac{2}{q\epsilon_{Si} N_d} \tag{16}$$

Therefore:

$$N_d = -\frac{2}{q\epsilon_{Si}\alpha} = 8.95 \cdot 10^{15} \left[cm^{-3}\right] \tag{17}$$

With Eq. 15, we can also see that the Mott-Schottky plot will intersect the zero at $\phi_b = V_0$, referencing the built-in potential formula:

$$N_a = \frac{n_i^2}{N_d} e^{V_0 \frac{q}{kT}} = 5.75 \cdot 10^{18} \left[cm^{-3}\right] \tag{18}$$

Finally with N_a and N_d we can validate our hypothesis:

$$(N_a) \quad 5.75 \cdot 10^{18} \gg 8.95 \cdot 10^{15} \quad (N_d) \tag{19}$$

To Go Further

A similar reasoning applies in the case of asymmetric doping where $N_d \gg N_a$. In this case, the slope will be:

$$\alpha = -\frac{2}{q\epsilon_{Si}N_a} \tag{20}$$

Exercise 03

As an electrical engineer, you are tasked with designing a VHF transmitter operating on the guard frequency (reserved for aircraft emergencies) at $f = 121.5\,[MHz]$. To achieve this, you decide to use the depletion capacitance of a diode (used as a varicap) to select the frequency in a simple LC oscillator.

The selected diode is a silicon diode with uncompensated doping, operating at room temperature, with the following dopant concentrations: $N_A = 4 \cdot 10^{16}\,\left[cm^{-3}\right]$ and $N_D = 8 \cdot 10^{15}\,\left[cm^{-3}\right]$ for the p-type and n-type regions, respectively. The p-n junction of the diode has a total length of $L_D = 900\,[\mu m]$ and a circular cross-section with an area of $A = 1\,\left[mm^2\right]$.The chosen inductance is $L = 12\,[nH]$.

Find the bias voltage required to tune the transceiver to the correct frequency.

Hint: As a reminder, the resonance frequency of an LC circuit is given by:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{21}$$

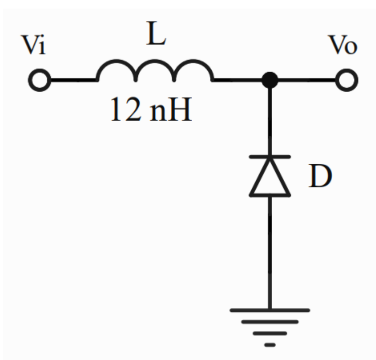

Figure 2: Basic LC resonator schematic.

Solution

This exercise is straightforward. First, we determine the capacitance required to achieve the desired frequency using the LC resonator formula:

$$C = \frac{1}{(2\pi f)^2 L} = 143\,[pF] \tag{22}$$

Next, we use the depletion capacitance formula from Eq. 9:

$$C = \frac{\epsilon}{d} = \frac{\epsilon_{Si}}{\sqrt{\frac{2\epsilon_{Si}(N_a+N_d)}{qN_aN_d}(\phi_b - V)}} \tag{23}$$

Since the dopant concentrations are relatively close to each other, we cannot apply the approximations used in the previous exercise (Eq. 11).

Until now, we have loosely used the term depletion capacitance, but if we take a closer look at the formula, this capacitance is actually normalized by area (per $\left[cm^2\right]$). In reality, we have been discussing depletion capacitance density rather than total capacitance. To obtain the actual depletion capacitance of the diode, we must multiply by the diode's area:

$$C_j\,[F] = C_j'\left[\frac{F}{cm^2}\right] \cdot A\left[cm^2\right] \tag{24}$$

From the depletion capacitance density formula, we see that we need the built-in potential:

$$\phi_B = \frac{kT}{q}\ln\left(\frac{N_aN_d}{n_i^2}\right) = 724\,[mV] \tag{25}$$

Finally, we rearrange the equation to solve for the required bias voltage:

$$V_B = \phi_b - \left(\frac{A}{C}\right)^2 \frac{q\epsilon_{Si}}{2}\frac{N_aN_d}{N_a+N_d} = -1.98\,[V] \tag{26}$$